THE UNSTEADY SPATIAL LAMINAR BOUNDARY LAYER IN MAGNETOHYDRODYNAMICS

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The equations of the unsteady plane laminar magnetohydrodynamic boundary layer have been solved by several authors ([1-4], for instance). Below we solve some problems for the following cases of spatial flows.

1. The boundary layer on an infinitely long cylinder to which a translational velocity and rotation are instantaneously imparted.

2. The boundary layer on a sideslipping wing which has begun to move with constant velocity.

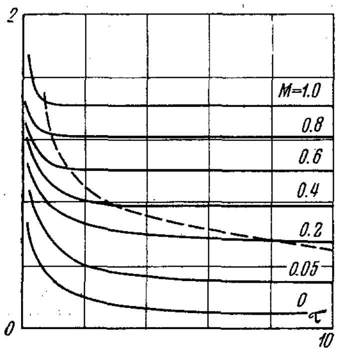

Fig. 1

1. BOUNDARY LAYER ON CYLINDER ROTATING IN AN AXIAL FLOW.

A) The equation of motion of a viscous fluid in the presence of a magnetic field normal to the surface of a body can be written in the approximate form [1]

$$\frac{dV}{dt} = -\rho^{-1}\nabla p + \nu\nabla^2 V - \sigma\rho^{-1}B_0^2 V. \qquad (1.1)$$

Where V is the velocity vector, p is the pressure, ρ is the density B_0 is the magnetic induction, σ is the electrical conductivity of the fluid, ν is the kinematic viscosity, and t is the time.

Assume that the thickness of the boundary layer is small in comparison with the cylinder radius. Then, for the case under consideration with a constant translational velocity U_0 and a constant angular velocity ω the projections of Eq. (1.1) onto the axial and circular direction lines when the magnetic field is fixed relative to the fluid (case a) will be

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} - \sigma\rho^{-1}B_0^2 u, \quad \frac{\partial v}{\partial t} = \nu\frac{\partial^2 v}{\partial z^2} - \rho\sigma^{-1}B_0^2 v. \qquad (1.2)$$

Here u and v are the velocity components in the boundary layer along the cylinder axis and in the circular direction; z is the coordinate measured along the perpendicular from the cylinder surface.

If the magnetic field is rigidly connected with the cylinder (case b), then

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial z^2} - \sigma\rho^{-1}B_0^2(u - U_0),$$
$$\frac{\partial v}{\partial t} = \nu\frac{\partial^2 v}{\partial z^2} - \sigma\rho^{-1}B_0^2(v - \omega r_0), \qquad (1.3)$$

where r_0 is the cylinder radius.

Equations (1.2) with corresponding boundary conditions are similar to those for the boundary layer on an infinite flat plate. Their solution can be found in [1].

B) If the thickness of the boundary layer is comparable to the cylinder radius, Eqs. (1.2) are no longer valid. We write the equations for case a in a cylindrical coordinate system

$$\frac{\partial u}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \sigma\rho^{-1}B_0^2 u,$$
$$\frac{\partial v}{\partial t} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - \frac{v}{r^2}\right] - \sigma\rho^{-1}B_0^2 v. \qquad (1.4)$$

The boundary and initial conditions are

$$u = U_0, \quad v = \omega r_0 \quad \text{when } r = r_0 \quad t > 0,$$
$$u \to 0, \quad v \to 0 \quad \begin{cases} \text{as } r \geqslant r_0 & t = 0, \\ \text{as } r \to \infty, & t \geqslant 0. \end{cases}$$

Converting (1.4) and (1.5) to the dimensionless variables

$$u^0 = \frac{u}{U_0}, \quad v^0 = \frac{v}{\omega r_0}, \quad \tau_0 = \frac{\nu t}{r_0^2}, \quad r^0 = \frac{r}{r_0}, \qquad (1.5)$$

and henceforth omitting the subscripts 0 in the dimensionless quantities, we obtain

$$\frac{\partial u}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - M^2 u,$$
$$\frac{\partial v}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - (r^{-2} + M^2)\, v, \quad M = \left(\frac{\sigma B_0^2 r_0^2}{\nu\rho}\right)^{1/2}, \qquad (1.6)$$

$$u = 1, \quad v = 1 \quad \text{when } r = 1, \quad \tau > 0,$$
$$u \to 0, \quad v \to 0 \quad \text{when} \begin{cases} r \geqslant 1, & \tau = 0, \\ r \to \infty, & \tau \geqslant 0. \end{cases} \qquad (1.7)$$

where M is the Hartmann number.

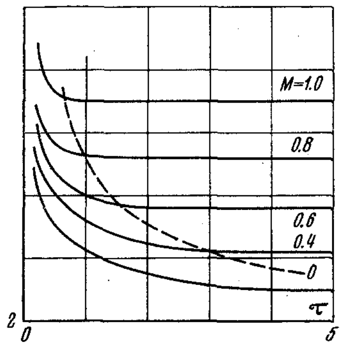

Fig. 2

To solve Eqs. (1.6) we use the Laplace transformation

$$U = \int_0^\infty e^{-s\tau} u\, d\tau, \qquad V = \int_0^\infty e^{-s\tau} v\, d\tau.$$

From (1.6) we obtain a system of Bessel equations for the images of U and V:

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} = (s + M^2)\, U,$$
$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = (s + M^2 + r^{-2})\, V, \qquad (1.8)$$

with boundary conditions

$$U = 1, V = 1 \quad \text{when } r = 1,$$
$$U \to 0, V \to 0 \quad \text{as } r \to \infty. \qquad (1.9)$$

Since images must become zero at infinity, from the two particular solutions of each of Eqs. (1.8), presented in the form of Bessel functions of imaginary argument of zero order for (1.8.1) and of first order for (1.8.2), we must use only the one which contains the MacDonald function, i.e.,

$$U = AK_0(r\sqrt{s+M^2}), \qquad V = BK_1(r\sqrt{s+M^2}). \qquad (1.10)$$

Determining the constants of integration A and B from the first boundary condition (1.9), we obtain

$$U = \frac{K_0(r\sqrt{s+M^2})}{K_0(\sqrt{s+M^2})}, \qquad V = \frac{K_1(r\sqrt{s+M^2})}{K_1(\sqrt{s+M^2})}. \qquad (1.11)$$

The originals u and v are found from the inversion formulas

$$u = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{K_0(r\sqrt{s+M^2})}{K_0(\sqrt{s+M^2})} e^{s\tau}\frac{ds}{s},$$

$$v = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{K_1(r\sqrt{s+M^2})}{K_1(r\sqrt{s+M^2}} e^{s\tau}\frac{ds}{s}. \qquad (1.12)$$

Integrating (1.12) we reduce the problem to quadratures

$$u = \frac{K_0(rM)}{K_0(M)} + \frac{2}{\pi}\int_0^\infty \frac{I_0(r\alpha)N_0(\alpha) - I_0(\alpha)N_0(r\alpha)}{I_0^2(\alpha)+N_0^2(\alpha)} \times$$

$$\times e^{-(\alpha^2+M^2)\tau}\frac{\alpha\,d\alpha}{\alpha^2+M^2}, \qquad (1.13)$$

$$v = \frac{K_1(rM)}{K_1(M)} + \frac{2}{\pi}\int_0^\infty \frac{I_1(r\alpha)N_1(\alpha) - I_1(\alpha)N_1(r\alpha)}{I_1^2(\alpha)+N_1^2(\alpha)} \times$$

$$\times e^{-(\alpha^2+M^2)\tau}\frac{\alpha\,d\alpha}{\alpha^2+M^2}. \qquad (1.14)$$

The friction coefficients C_f and the friction moment C_M are given by

$$C_f = \frac{2\mu}{\rho U_0^2}\left(\frac{\partial u}{\partial r}\right)_{r=r_0} = -\frac{2}{R_1}\left[\frac{MK_1(M)}{K_0(M)} + \right.$$

$$\left. + \frac{2}{\pi^2}\int_0^\infty \frac{\exp[-(\alpha^2+M^2)\tau]}{I_0^2(\alpha)+N_0^2(\alpha)}\frac{\alpha\,d\alpha}{\alpha^2+M^2}\right], \quad R_1 = \frac{U_0 r_0 \rho}{\mu}, \qquad (1.15)$$

$$C_M = \frac{4\pi\mu}{\rho\omega^2 r_0^2}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)_{r=r_0} =$$

$$= -\frac{4\pi}{R_2}\left[\frac{M}{2}\frac{K_2(M)+K_0(M)}{K_1(M)} + \right.$$

$$\left. +1 + \frac{2}{\pi^2}\int_0^\infty \frac{\exp[-(\alpha^2+M^2)\tau]}{I_1^2(\alpha)+N_1^2(\alpha)}\frac{\alpha\,d\alpha}{\alpha^2+M^2}\right], \quad R_2 = \frac{\omega r_0^2\rho}{\mu}. \qquad (1.16)$$

As $M \to 0$, Eqs. (1.14) and (1.16) become the solutions given in [5] for ordinary hydrodynamics.

For case b the procedure is similar to that given.

C) The values of u, v, C_f, and C_M can be found by numerical methods. However, they will be particular solutions which are not suitable enough for practical application. We will attempt to obtain approximate formulas for C_f and C_M.

Numerical calculations showed that a sufficiently accurate approximation is

$$[I_i^2(\alpha) + N_i^2(\alpha)]^{-1} \approx C_i\alpha \quad (i = 0, 1);$$

$$C_0 = 1.686, \quad C_1 = 1.244. \qquad (1.17)$$

Then the quadratures in (1.15) and (1.16) are calculated [6]

$$C_i\int_0^\infty \frac{\exp[-(\alpha^2+M^2)\tau]}{\alpha^2+M^2}\alpha^2\,d\alpha =$$

$$= \frac{C_i}{2}\left\{\left(\frac{\pi}{\tau}\right)^{1/2}\exp(-M^2\tau) - \pi M\,[1-\operatorname{erf}(M\sqrt{\tau})\right\}. \qquad (1.18)$$

Hence, in view of (1.18), we recommend replacement of (1.15) and (1.16) by the following approximate formulas:

$$C_f = -\frac{2}{R_1}\left[\frac{MK_1(M)}{K_0(M)} + \frac{C_0}{\pi^2}\left\{\left(\frac{\pi}{\tau}\right)^{1/2}\times\right.\right.$$

$$\left.\left.\times\exp(-M^2\tau) - \pi M\,[1-\operatorname{erf}(M\sqrt{\tau})]\right\}\right], \qquad (1.19)$$

$$C_M = -\frac{4\pi}{R_2}\left[\frac{M}{2}\frac{K_2(M)+K_0(M)}{K_1(M)} + 1 + \right.$$

$$\left. + \frac{C_1}{\pi^2}\left\{\left(\frac{\pi}{\tau}\right)^{1/2}\exp(-M^2\tau) - \pi M\,[1-\operatorname{erf}(M\sqrt{\tau})]\right\}\right]. \qquad (1.20)$$

It follows from (1.19) and (1.20) that with increase in M the time required to attain the limit values of C_f and C_M is reduced. Figures 1 and 2 show the changes in C_f and C_M for several values of M. The regions of steady and unsteady regimes in Figs. 1 and 2 are separated by dashed lines. These boundaries are drawn on the assumption that the regime is steady when C_f and C_M differ from their limit values by no more than 1%.

It is an interesting fact that in ordinary hydrodynamics $u(r,\tau) \to 1$ as $\tau \to \infty$ [see (1.13)], i.e., the fluid moves together with the cylinder as a solid body. When $M \neq 0$, there are limiting velocity profiles differing from u = const. To illustrate the effect of the number M on the velocity profiles in steady regimes, Figs. 3 and 4 show the results for calculations of the relationship between u/U_0 and r/r_0, where r_0 is the cylinder radius. It is clear that with increase in M the thickness of the boundary layer decreases, i.e., the magnetic field "presses" the boundary layer against the cylinder.

2. BOUNDARY LAYER ON SIDESLIPPING WING.

A) Let a sideslipping wing (sideslip angle β) situated in a transverse magnetic field be instantaneously given a translational velocity W_0. Since the flow characteristics are independent of the coordinate along the wing span, the equations of motion and continuity take the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho}u, \qquad (2.1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho}v,$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \qquad (2.3)$$

where u, v, and w are the velocity components in the direction of the chord, the span, and the normal to the surface of the wing, while x and z are the coordinates measured along the chord (along the normal to the generatrix of the wing) and along the normal to the wing surface, respectively.

Equations (2.1) and (2.3) are not associated with (2.2) and, hence, they can be solved independently of the latter. This system is similar to that describing the development of a boundary layer on a body in a plane flow [2].

Equations (2.1) and (2.3) are solved with the following boundary and initial conditions:

$$u = v = w = 0 \quad \text{when} \begin{cases} z \geqslant 0, & t = 0, \\ z = 0, & t \geqslant 0, \end{cases}$$

$$u \to U(x), \quad v \to V(x) \quad \text{as} \quad z \to \infty,\ t > 0. \qquad (2.4)$$

From the boundary conditions (2.4) and Eqs. (2.1) and (2.3) as $z \to \infty$ we have

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = U\left(\frac{dU}{dx} + \frac{\sigma B_0^2}{\rho}\right), \quad U\frac{dV}{dx} = -\frac{\sigma B_0^2}{\rho}V. \tag{2.5}$$

We note that U corresponds exactly with the velocity distribution outside the boundary layer in the case of a plane flow. We will assume

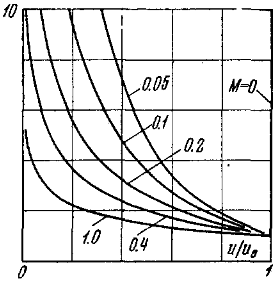

Fig. 3

it to be a known function of the coordinate x. Then from (2.5.2) and the condition that in the incoming flow

$$V_\infty = W_0 \sin\beta, \quad U_\infty = W_0 \cos\beta, \tag{2.6}$$

it follows that

$$V = V_\infty \exp\left[-\frac{\sigma B_0^2}{\rho}\int_0^x \frac{dx}{U(x)}\right]. \tag{2.7}$$

We introduce the symbols

$$u^\circ = \frac{u}{U_\infty}, \quad v^\circ = \frac{v}{U_\infty}, \quad U^\circ = \frac{U}{U_\infty}, \quad V^\circ = \frac{V}{U_\infty},$$

$$w^\circ = \frac{w}{U_\infty}\sqrt{R}, \quad x^\circ = \frac{x}{b}, \quad z^\circ = \frac{z}{b}\sqrt{R},$$

$$R = \frac{U_\infty b}{\nu}, \quad m = \frac{\sigma B_0^2 b}{\rho U_\infty} = \frac{M^2}{R}, \quad \tau = \frac{U_\infty t}{b} \tag{2.8}$$

where b is the chord of the wing.

Then, using (2.5.1) we can write Eqs. (2.1)-(2.4) and (2.7) in dimensionless form (superscripts ° omitted)

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = U\left(\frac{\partial U}{dx} + m\right) + \frac{\partial^2 u}{\partial z^2} - mu,$$

$$\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} - mv, \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

$$V(x) = \operatorname{tg}\beta \exp\left[-m\int_0^x \left[\frac{dx}{U(x)}\right]\right]; \tag{2.9}$$

$$u = v = w = 0 \quad \text{when} \begin{cases} z \geqslant 0, & \tau = 0 \\ z = 0, & \tau \geqslant 0 \end{cases},$$

$$u \to U(x), \quad v \to V(x) \quad \text{as} \quad z \to \infty, \ \tau > 0. \tag{2.10}$$

As in [2], we will seek the solution of system (2.9) and (2.10) in the form

$$\psi = 2\sqrt{\tau}\,[f_0(x,\eta) + \tau f_1(x,\eta) + \tau^2 f_2(x,\eta) + \ldots],$$

$$u = \frac{\partial\psi}{\partial z} = \frac{\partial f_0}{\partial\eta} + \tau\frac{\partial f_1}{\partial\eta} + \tau^2\frac{\partial f_2}{\partial\eta} + \ldots,$$

$$w = -\frac{\partial\psi}{\partial x} = -2\sqrt{\tau}\left(\frac{\partial f_0}{\partial x} + \tau\frac{\partial f_1}{\partial x} + \tau^2\frac{\partial f_2}{\partial x} + \ldots\right),$$

$$v = g_0(x,\eta) + \tau g_1(x,\eta) + \tau^2 g_2(x,\eta) + \ldots, \quad \left(\eta = \frac{z}{2\sqrt{\tau}}\right). \tag{2.11}$$

Substituting (2.11.1) into (2.9) and equating the coefficients of equal powers of τ, we obtain

$$\frac{\partial^3 f_i}{\partial\eta^3} + 2\eta\frac{\partial^2 f_i}{\partial\eta^2} - 4i\frac{\partial f_i}{\partial\eta} = 2\Pi_{1i},$$

$$\frac{\partial^2 g_i}{\partial\eta^2} + 2\eta\frac{\partial g_i}{\partial\eta} - 4ig_i = 2\Pi_{2i}, \tag{2.12}$$

$$\Pi_{1i} = 2\delta\left[-\gamma U\left(\frac{dU}{dx} + m\right) + m\frac{\partial f_{i-1}}{\partial\eta} - \sum_{\substack{j,\,k=0\\ j+k=i-1}}^{i-1}\left(\frac{\partial f_j}{\partial x}\frac{\partial^2 f_k}{\partial\eta^2} - \frac{\partial f_j}{\partial\eta}\frac{\partial^2 f_k}{\partial x\partial\eta}\right)\right], \tag{2.13}$$

$$\Pi_{2i} = 2\delta\left[mg_{i-1} - \sum_{\substack{j,\,k=0\\ j+k=i-1}}^{i-1}\left(\frac{\partial f_j}{\partial x}\frac{\partial g_k}{\partial\eta} - \frac{\partial f_j}{\partial\eta}\frac{\partial g_k}{\partial x}\right)\right],$$

$$\delta = 0 \text{ when } i = 0, \ \delta = 1 \text{ when } i \geqslant 1,$$

$$\gamma = 1 \text{ when } i = 1, \ \gamma = 0 \text{ when } i \geqslant 2. \tag{2.14}$$

The boundary and initial conditions are

$$f_i = \frac{\partial f_i}{\partial\eta} = g_i = 0 \quad \text{when } \eta = 0$$

$$\frac{\partial f_0}{\partial\eta} \to U(x), \ g_0 \to V(x), \ \frac{\partial f_i}{\partial\eta} \to 0, \ g_i \to 0\ (i \geqslant 1) \ \text{as} \ \eta \to \infty. \tag{2.15}$$

It follows from (2.12)-(2.15) that for i = 0

$$\frac{1}{U(x)}\frac{\partial f_0}{\partial\eta} = \frac{1}{V(x)}g_0 = \operatorname{erf}\eta. \tag{2.16}$$

Taking into account the results of [2], for i = 1 we can write

$$\frac{\partial f_1}{\partial\eta} = L[\eta, \Pi_{1i}], \quad g_1 = L[\eta, \Pi_{2i}]. \tag{2.17}$$

The function L has the properties

$$L[\eta, \varphi] \leqslant 0 \ (\varphi \geqslant 0, \ 0 < \eta < \infty)$$

$$L[\eta, k_1\varphi_1 + k_2\varphi_2] =$$

$$= k_1 L[\eta, \varphi_1] + k_2 L[\eta, \varphi_2] \ (k_1, k_2 - \text{const}),$$

and the following form:

$$L[\eta,\varphi] = -\frac{1}{2}\sqrt{\pi}(1-2\eta^2)\left\{C\operatorname{erf}\eta + \int_0^\eta \varphi(\eta)\left[(1-2\eta^2)\exp\eta^2\operatorname{erf}\eta + \frac{2\eta}{\sqrt{\pi}}\right]d\eta - \operatorname{erf}\eta\int_0^\eta \varphi(\eta)(1-2\eta^2)\exp\eta^2\,d\eta\right\} - C\eta\exp(-\eta^2) + \eta\exp(-\eta^2)\int_0^\eta \varphi(\eta)(1-2\eta^2)\exp\eta^2\,d\eta$$

$$\left(C = \int_0^\infty \varphi(\eta)G(\eta)\,d\eta, \ G(\eta) = (1-2\eta^2)\operatorname{erfc}\eta\exp\eta^2 - \frac{2\eta}{\sqrt{\pi}}\right). \tag{2.19}$$

The equations for approximations of higher order can also be reduced to ordinary equations, which can be solved in quadratures.

B) It follows from the results of [2] that:

$$\left(\frac{\partial^2 f_1}{\partial\eta^2}\right)_{\eta=0} = -4\int_0^\infty G(\eta)\Pi_{11}(\eta)\,d\eta, \quad \left(\frac{\partial g_1}{\partial\eta}\right)_{\eta=0} = -4\int_0^\infty G(\eta)\Pi_{21}(\eta)\,d\eta. \tag{2.20}$$

Again from [2]

$$\left(\frac{\partial^2 f_1}{\partial\eta^2}\right)_{\eta=0} = \frac{2}{\sqrt{\pi}}\left[m + U'\left(1 + \frac{4}{3\pi}\right)\right]U. \tag{2.21}$$

Substituting Π_{21} from (2.14) into (2.20.2) and using (2.16) we can write

$$\left(\frac{\partial g_1}{\partial \eta}\right)_{\eta=0} = U'VM_1 + UV'N + mVC_1, \qquad (2.22)$$

$$M_1 = 4\int_0^\infty G(\eta)\varkappa(\eta)\,d\eta, \quad \varkappa(\eta) = (\operatorname{erf}\eta)'\left(\int_0^\infty \operatorname{erf}\eta\,d\eta\right),$$

$$N = -4\int_0^\infty (\operatorname{erf}\eta)^2 G(\eta)\,d\eta, \quad C_1 = -4\int_0^\infty \operatorname{erf}\eta G(\eta)\,d\eta. \qquad (2.23)$$

After calculating the integrals in (2.23) we find that

$$M_1 = C_1 = \frac{2}{\sqrt{\pi}}, \quad N = \frac{8}{3}\pi^{-\frac{3}{2}}. \qquad (2.24)$$

In view of (2.22), (2.24), and (2.16), the coefficients of friction on the wing surface are

$$c_X = \frac{2}{\sqrt{\pi R}}\frac{1}{\sqrt{\tau U}}\left\{1 + \tau\left[m + U'\left(1 + \frac{4}{3\pi}\right)\right] + \cdots\right\}, \qquad (2.25)$$

$$c_Z = \frac{2}{\sqrt{\pi R}}\frac{V}{U}\frac{1}{\sqrt{\tau U}}\left\{1 + \tau\left[m + U'\left(1 + \frac{4}{3\pi}\frac{U}{V}\frac{V'}{U'}\right)\right] + \cdots\right\}. \qquad (2.26)$$

Formula (2.25) was obtained in [2]. It follows that the instant at which the boundary layer separates (if this happens)—found with only the first two approximations—is determined from (2.25) by

$$\tau^* = [-m - U'(1 + {}^4/_3\pi^{-1})]^{-1}. \qquad (2.27)$$

Equations (2.25) and (2.26) are written most simply for the case of flow over a sliding flat plate in a homogeneous transverse magnetic field. It follows from (2.5.1) and (2.10.1) that in this case:

$$U = 1 - mx, \qquad V = \operatorname{tg}\beta(1 - mx). \qquad (2.28)$$

Substituting (2.28) into (2.25) and (2.26), accurate to τ^2 we obtain

$$c_X = \frac{c_Z}{\operatorname{tg}\beta} = \frac{2}{\sqrt{\pi R}}\frac{2}{\sqrt{\tau(1 - mx)}}\left(1 - \frac{4m}{3\pi}\tau + \cdots\right), \qquad (2.29)$$

or with the same accuracy

$$c_Z = \operatorname{tg}\beta\, c_X. \qquad (2.30)$$

Result (2.30) agrees (with the accuracy indicated above) with the result obtained for steady flow over a sliding flat plate in ordinary hydrodynamics ([7], for instance).

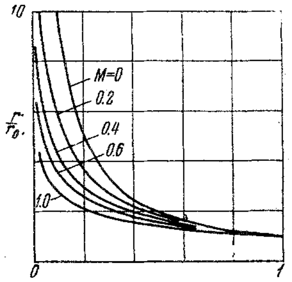

Fig. 4

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